The Astronomy and Mathematics History of Ancient India

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ABSTRACT

Ancient India's history in mathematics and astronomy is proof of the great intellectual accomplishments of this advanced society. Indian academics have been making significant contributions to various subjects since antiquity, leaving a lasting impression on the annals of human knowledge. The ancient Indians made accurate astronomical equipment like the sundial and astrolabe and developed advanced observing methods. The "Aryabhatiya," a foundational book that addressed a variety of astronomical themes, including the heliocentric model of the solar system, was written in the fifth century CE by the well-known Aryabhata. With the introduction of the decimal system, which established the groundwork for contemporary algebra and arithmetic, mathematics flourished in ancient India. In the seventh century CE, the Brahmagupta wrote the "Brahmasphutasiddhanta," which delves into complex mathematical ideas and introduces zero as a placeholder and solutions to quadratic equations.

Keywords: *Astronomy; Mathematics; History; Ancient India*

INTRODUCTION

The intricate web of knowledge that ancient India constructed from astronomy and mathematics shines like a light. Throughout the subcontinent's history, intellectuals have made incredible achievements in fields as diverse as stargazing and the study of complex numbers and geometric patterns. Both fields met in ancient Indian philosophy, which changed regional science and made an everlasting impression on scientific history worldwide. Across the long swath of history, Indian astronomy has its origins in the Vedic era, when the Rig Vedica hymns demonstrate a primitive knowledge of the motions of the heavens. A book connected with the Vedic literature and devoted to the study of astronomy, the Vedanga Jyotisha, is one example of how the interest with the heavens persisted through succeeding epochs. The mathematical exactness used to ideas like timekeeping and calendar computations demonstrates an early merging of mathematics and astronomy, marking a dramatic transition from observational research to systematic investigation.[1]

Aryabhata, a famous Indian mathematician and astronomer from the 5th century CE, is largely responsible for one of the greatest accomplishments in ancient Indian astronomy. In the Aryabhatiya, his masterwork, Aryabhata not only established precise ways of determining the locations of the planets, but he also presented the idea of zero, a fundamental building block of mathematics that would transform numerical systems around the globe. Prior to the rise of other greats like Brahmagupta and Bhaskara, Aryabhata's groundbreaking work in Indian astronomy had already begun to take shape [2]. In the famous work Siddhanta Shiromani, written by the 12th-century mathematician and astronomer Bhaskara II, the integration of mathematics and astronomy reached unprecedented levels. Part one of this massive four-part book dealt with algebraic methods, part two with planetary motion, and part three with mathematical ideas like solving quadratic equations. Fibonacci and other European mathematicians owe a debt to Bhaskara II, whose influence went well beyond the subcontinent [3].

Ancient India is the birthplace of the decimal system, an essential part of contemporary mathematics. Brahmagupta laid forth the principles of mathematical operations, including the ground-breaking concept of zero as a placeholder, in his influential treatise

Brahmasphutasiddhanta. This idea was a game-changer in mathematical thought; it influenced the growth of Western mathematics and made its way to the Arab world via translations. The great achievements of ancient Indian mathematicians and astronomers extended well beyond the realm of theoretical treatises. The integration of theoretical understanding with actual observations was demonstrated through practical implementations, such as the 18th-century observatory built at Jaipur by Maharaja Sawai Jai Singh II. Monuments to the scientific ability of ancient Indian astronomers, these observatories stand with their accurate apparatus.[4]

HISTORY

Even earlier than that, during the time of the Indus Valley civilization, there were some early attempts at astronomy. The Vedas include certain cosmological ideas, as well as thoughts of the motion of celestial bodies and the passage of time. Among the first works written in India is the Rig Veda. In relation to the solar calendar, the Rig Veda 1-64-11 & 48 depicts time as a wheel with 12 spokes and 360 days, leaving a residual of 5. The early history of astronomy was closely tied to religious practice, as is true in other cultures; this is because religious rituals required precise spatial and temporal measurements, which could only be met by careful astronomical observation [5]. Therefore, complex mathematics and elementary astronomy are covered in the Shulba Sutras, which are books that are devoted to the building of altars. Details on the Sun, Moon, nakshatras, and the lunisolar calendar are included in Vedanga Jyotisha, one of the oldest known Indian astronomical works. For ceremonial reasons, the Vedanga Jyotisha lays forth guidelines for following the Sun's and Moon's planetary movements. An era, or yuga, in Hinduism is defined as five solar years, sixty-seven lunar sidereal cycles, one thousand eight hundred thirty days, one thousand eight hundred thirty-five sidereal days, and sixty-two synodic months. [6]

In the fourth century BCE, after Alexander the Great's conquests, Greek astrological concepts started making their way into India. The Yavanajataka and Romaka Siddhanta are two examples of literature that show how the Indo-Greek tradition impacted astronomy in the early Common Era [7]. Astronomers who came after this time period noted the presence of many siddhantas, including the Surya Siddhanta. Their contents do not exist as they were not written documents but rather an oral transmission of information. Aryabhata obtained the text that is now known as Surya Siddhanta during the Gupta dynasty [8].

Beginning in the late Gupta period, during the 5th to 6th century, Indian astronomy enters its classical period. This method for approximating the meridian direction from any three shadow positions using a gnomon is described in the Pañcasiddhāntikā by Varāhamihira (505 CE). The elliptical rather than the circular motion of the planets was accepted by Aryabhata's time. Different time units were defined, planetary motion models (eccentric and epicyclic), and planetary longitude adjustments for different terrestrial sites were covered as well [9].

ASTRONOMY IN ANCIENT INDIA

The notion that ancient Indian astronomers were keen watchers of the skies is supported by a great number of sources throughout history. The Vedas, which are considered to be the most important holy scriptures in Hinduism, contain a discussion on the movement of the sky. However, following texts like as the Brahmanas and the Puranas detailed astronomical observations and computations that were more exact than those described in the literature.[10]

The "Surya Siddhanta," an old piece of Sanskrit literature, is particularly noteworthy among these texts since it provides a comprehensive description of the phases of the moon and the sun. Both the "Arya-Siddhanta" and the "Aryabhatiya" written by Aryabhata are considered to be foundational writings that investigate the principles of astronomy. These books may provide you with accurate yearly calendars, planetary positions, and eclipse periods all in one convenient location [11].

In Indian astronomy, the concept of "yugas" or cosmic cycles was a fundamental component. According to this theory, the universe goes through a series of cyclical stages of creation, maintenance, and annihilation.

MATHEMATICS IN ANCIENT INDIA

Mathematicians from ancient India produced significant contributions that were crucial in laying the groundwork for subsequent mathematical advancements throughout history. The Sulba Sutras, which are considered to be a work of Vedic literature, are considered to be the first known mathematical books. The Vedic scriptures include geometric ideas and procedures for constructing altars for the purpose of performing Vedic ceremonies.[12]

In his book "Brahmasphutasiddhanta," the great mathematician Brahmagupta made substantial contributions to the field of algebra. These contributions included the introduction of zero as a number and the rules for arithmetic operations that use zero. In addition, the book "Aryabhatiya" by Aryabhata discusses a variety of mathematical ideas, such as trigonometry and algebraic equations.

One of the most important developments in Indian mathematics was the introduction of the decimal system, which included the idea of zero. It is common practice to refer to the numeric system that uses the digits 0-9 as the Hindu-Arabic numeral system, which highlights the fact that it originated in India [13].

VEDIC MATHEMATICS: THE SULBA SŪTRAS

The Vedic Hindu, on the other hand, achieved remarkable and unmatched levels of completion in his pursuit of Para-vidyā ("Supreme knowledge"), Satyasya Satyam ("Truth of truths", "Absolute Truth"), and Aparā-vidyā ("inferior knowledge", "relative truths"), which encompasses a wide range of arts and sciences. [14]

When mathematics was first taking shape, it largely followed two interconnected traditions: (i) geometry and (ii) algebra and mathematics. Both of these major branches of mathematics were highly valued by ancient Indian civilizations, which existed before Greece. Ancient civilizations that followed the computational tradition included the Babylonians and the Egyptians. Renowned math historian and algebraist A. Seidenberg found that the creators of the Rg-Vedic ceremonies were the ones who first introduced complex mathematical concepts.

The Sulba Sūtras of Baudhāyana, Apastamba, and Kātyāyana are the earliest known mathematical books. They are a component of the literature of the sūtra period in the later Vedic age. Although older dates have been proposed by certain experts, the conventional wisdom was that the Sulba Sūtras were written about 800 BC. However, the mathematical wisdom preserved in these sūtras (aphorisms) dates back considerably farther in time; the writers of the Sulba texts stress that they were only restating information that was previously known to the early Vedic era composers of the Brahmaṇas and Samhitas [15].

The Sulba Sūtras include a collection of mathematical findings that have been utilised for the creation of several exquisite Vedic fire-altars from the beginning of civilization. The careful construction of an altar was required because of the great symbolic importance of the object. A few of these brick-altars include elaborate designs; for example, you may find ones that portray a chariot-wheel with spokes, a tortoise with its head and legs spread, or even a falcon in flight with its curled wings! The Satapatha Brāhmaṇa (c. 2000 BC; vide [2]) provides a highly developed description of the fire-altar constructions. The earlier Taittiriya Samhita (c. 3000 BC; vide [2]) also makes reference to some of these constructions, but the sacrificial fire-altars are referred to in the even older Rg-Vedic Samhitas, the oldest section of the Vedic literature that is still extant, although without explicit construction. The later Sulba Sūtras and the Taittiriya Samhita both include identical accounts of fire-altars.[16]

In Sulba mathematics, one must have an in-depth familiarity with the algebraic and geometric features of regular shapes like triangles, squares, rectangles, circles, parallelograms, trapezia, and the like. Two fundamental results with long-lived implications for plane geometry are (i) the "Pythagoras theorem" and (ii) the characteristics of comparable figures. The Pythagorean theorem[3] and its geometric applications are laid forth in the Sulba Sūtras. These include the building of a square with an area equal to the sum or difference of two specified squares, a rectangle, or the sum of n squares, among other examples. Simple algebraic identities like $(a+b)^2= a^2+b^2+2ab$, $a^2-b^2= (a+b)(a-b)$, $ab = ((a+b)/2)^2-((a-b)/2)^2$, and $na^2 = ((n+1)/2)$ are implicitly used in these constructs. The expression $2a^2-((n-1)/2)^2a^2$ was written. In these, we see the combination of Euclidean geometry with the complex algebraic reasoning that he is known for. Interestingly, even after all these years, the Sulba construction of a square with an area equal to a certain rectangle is identical to the one described by Euclid! Algebraic and number-theoretic issues have geometric solutions. Several centuries prior to Pythagoras (580-495 BC) and Euclid (365-275 BC), Indians had achieved proficiency in essential parts of Euclidean geometry during the Vedic period of distant antiquity, as shown by their geometric algebra and geometry insights.

In contrast to the Sulba Sutras, which provide extensive treatment of both numerical and appropriate geometric components of Pythagoras' theorem, other ancient civilizations, such as the Babylonian, were aware of it. Seidenberg thoroughly examines this nuanced subject. Based on specific graphic representations in the Sulba Sūtras, a number of mathematicians and historians, including Burk, Hankel, Schopenhauer, Seidenberg, and van der Waerden, have deduced that the Sulba authors had evidence of geometrical conclusions, such as Pythagoras' theorem; Datta's seminal work ([2]) examines some of these details. A more detailed description of one of the proofs of Pythagoras theorem is provided by Bhāskara II (1150 AD), although it is simply deducible from the Sulba verses.[17]

In addition to their proficiency in geometry and algebraic geometry, the Vedic civilization excelled in computational mathematics. They were experts in surd and fraction arithmetic, found reasonable approximations to irrational numbers like 2's square root, and, of course, applied numerous noteworthy findings in mensuration.

Written completely in rhyme, the mathematical literature of ancient India, starting with the Sulba Sutras, is a remarkable achievement. In spite of the scarcity and transience of written materials, the custom of creating concise sūtras that could be readily memorised allowed for the oral transmission of certain essential knowledge to subsequent generations.

POST-VEDIC MATHEMATICS

The Greeks, who lived from 600 BC to 300 AD, laid the groundwork for modern mathematics with their axiomatic approach, built the grand structure of Euclidean geometry, established trigonometry, made remarkable progress in number theory, and revealed the inherent splendour, grace, and beauty of pure mathematics. Greek geometry advanced to higher geometries with the development of conic sections by Archimedes and Apollonius, building on the firm groundwork laid by Euclid. Along with several other groundbreaking contributions to mathematics and science, Archimedes brought integration to the table. Yet, following this illuminating period of Greek mathematics, the West essentially froze until the contemporary renaissance.

However, notably in mathematics, algebra, and trigonometry, the Indian contribution persisted energetically from the beginning all the way up to the 16th century AD. It was actually only in India and, to a lesser degree, China that one could discover a plethora of inventive and innovative mathematical activity for a few centuries following the fall of the Greeks. Modern academics held Indian mathematics in the highest respect. A Spanish monk named Vigila recorded this in a document from 976 AD [18].

Math, geometry, and other complex fields are off-limits to non-Indian concepts due to the Indians' exceptionally nuanced and perceptive minds. The fact that they use the same nine symbols to symbolise all numbers, no matter how big, is the strongest evidence of this.

In a brief allusion in a passage from the work of the esteemed Syrian astronomer-monk Severus Sebokht (662 AD), it is mentioned that the renown of Indian mathematics had spread to the banks of the Euphrates by the early 7th century: "I shall not now speak of the knowledge of the Hindus,... of their subtle discoveries in the science of astronomy—discoveries even more ingenious than those of the Greeks and Babylonians of their rational system of mathematics, or of their method of calculation which no words can praise strongly enough—I mean the system using nine symbols."

The Decimal Notation and Arithmetic

An invaluable gift, the decimal system, was bestowed upon the globe by India. When it comes to practicality and abstract reasoning, this remarkable, unnamed Indian creation is unparalleled. Two brilliant ideas—the concept of place value and the idea of zero as a digit—are the fundamental sources of decimal notation's potency. Using stunning images, G. B. Halsted demonstrated the significance of the place-value of zero:

The significance of the zero mark's invention cannot be overstated. The Hindu race, from whom it originated, is characterised by this practice of bestowing upon insubstantial things not just a physical location and a name, an image, a symbol, but also beneficial power. You might say it's the same as turning Nirvana into dynamos. When it comes to the overall progress of knowledge and power, no mathematical invention has been more effective than this one.

The decimal system is taught to youngsters at a young age all around the globe because to its seemingly simple nature. Its computational capability is its most valuable asset, and it also has an economy in the amount of symbols required and the space used by a written number. It also has the potential to readily represent arbitrarily big numbers. Decimalizing the twelve-digit Roman numeral (DCCCLXXXVIII) yields the simple number 888. The majority of the commonly used answers in elementary mathematics have their roots in India. This encompasses.[19]

the rules of operations with fractions and surds, neat, systematic, and straightforward methods of basic arithmetic operations (adding, subtracting, multiplying, dividing, taking squares and cubes, and extracting square and cube roots), as well as rules on ratio and proportion (such as the rule of three), and a number of business and related problems (such as income and expenditure, profit and loss, discounting, partnerships, calculating average impurities of gold, speeds and distances, and issues involving mixtures and cisterns, which are similar to those found in modern texts). Even as late as the fourteenth century AD, Europeans began using Indian techniques for lengthy divisions and multiplications. Because of our familiarity with fractional operations, we often fail to notice that they include concepts that the ancient Egyptians and Greeks, two of the most mathematically gifted civilizations in history, could not understand. Following the Renaissance, merchants placed a premium on the rule of three, which had been imported to Europe from the Arab world. The early European mathematicians spent a lot of work on this rule, and it became famous as the Golden Rule because of how useful it is in business presentations [20].

The benefit of the early discovery of the decimal notation—the key to all main notions in contemporary mathematics—was largely responsible for the proficiency and ability gained by the Indians in the foundations of mathematics. Äryabhaṭa, who lived in the 5th century AD, outlined contemporary techniques for finding square and cube roots by deftly using the concepts of place-value, zero, and the algebraic expansions of $(a + b)^2$ and $(a + b)^3$. It wasn't until the 16th century that these practices made it to Europe. Indians came up with a lot of clever ways to get the approximate square roots of non-square integers, in addition to the exact approaches [21].

Algebra

The axiomatization and subsequent development of geometry were accomplished by the Greeks, but it had already existed at the inception of Vedic ceremonies. No other ancient or mediaeval civilization could surpass the geometric heights attained by the Greeks during Apollonius' period (260-170 BC). However, advancements in "real" geometry quickly stalled. The singular jewels in the history of geometry are Brahmagupta's dazzling theorems (628 AD) on cyclic quadrilaterals, which span the centuries between Pappus (300 AD), the last great name in Greek geometry, to contemporary Europe. A fresh strategy, if not a whole new field of mathematics, was required for further advancement. The birth and growth of algebra as a field of study supplied this. A revival in geometry started in the early 17th century with the algebraization of geometry by Descartes and Fermat, although it didn't happen until the 16th century AD, when an algebra culture emerged in European mathematics. Europeans made tremendous advances in calculus and number theory beginning in the 17th century, which was largely due to the integration and improvement of algebra.[22]

Before literal or symbolic algebra was introduced in India, algebra was only implicit in the mathematics of a number of ancient civilizations. Symbolic algebra was already well-established in India as a separate subfield of mathematics by the time of Brahmagupta (628 AD) and Aryabhaṭa (499 AD). Algebra has progressed through various phases and is now an essential tool in many areas of modern mathematics in addition to its own distinct discipline. "Algebraisation of mathematics" was really quite popular in the twentieth century. For mathematicians, algebra is a source of technical prowess as well as elegance, simplicity, accuracy, and clarity. Brahmagupta (628 AD) and Bhāskara II (1150 AD), two prominent Indian mathematicians, firmly stated and confirmed the relevance of algebra, a newly-founded field, as we will see in the following sections. It is surprising how early the Indians understood the significance of algebra [23].

The Indians started using symbols in a systematic way to represent mathematical processes and unknown numbers. "Yu", "kṣa", "gu" and "bhā" were used to represent the four arithmetic operations, which are the Sanskrit words for addition, subtraction, multiplication and division, respectively. Similarly, "mu" or "ka" stood for mūla or karani, meaning root, and the initial letters of various colours were used to represent different unknown variables. The quick development of mathematics was greatly aided by the advent of symbolic representation. The Greek writings of Diophantus also display a primitive use of symbols, but algebraic formalism was fully developed in India. In their extensive study of equations, known as samikarana, the Indians accomplished many things: they introduced negative numbers and the rules for arithmetic operations involving zero and negative numbers; they found solutions to linear and quadratic equations; they provided formulae for arithmetic and geometric progression and identities involving summation of finite series; they applied various useful results on permutation and combinations, including the formulae for "P" and "C."

LEGACY AND INFLUENCE

i. Cultural Impact:

- **Astronomy in Religion and Rituals:** Ancient Indian religious and ceremonial activities placed a strong emphasis on astronomy. Temples and other places of worship were often oriented to the stars. Certain celebrations and rituals were thought to benefit from the alignment of the stars and planets.
- **Timekeeping Systems:** The astronomers of ancient India created complex methods of keeping track of time. The astronomically-based Indian calendar was vital in coordinating religious and agricultural pursuits. The lunar and solar aspects of the Hindu calendar show how closely related astronomy and cultural events are.

ii. Global Impact:

- **Transmission to the Islamic World:** The Islamic world gained access to Indian astronomy and mathematical expertise during the Middle Ages via translations and exchanges. Islamic Golden Age scholars like Al-Khwarizmi and Al-Biruni played a crucial role in maintaining and expanding upon this knowledge, which in turn impacted the evolution of Islamic mathematics and astronomy.[24]
- **Influence on European Renaissance:** During the Renaissance, Islamic academics helped spread mathematical concepts from India, which sparked a new wave of thinking in Europe. Ideas like algebra, zero, and the Indian number system revolutionised mathematics in Europe and facilitated progress in many fields of science.

iii. Contemporary Relevance:

- **Mathematics and Computer Science:** Zero and the Indian number system are cornerstones of contemporary mathematics. It was foundational to the growth of two modern mathematical pillars, algebra and calculus. Furthermore, the foundational computer science concept of the binary number system may be traced back to ancient Indian mathematical philosophy.
- **Astronomy and Space Exploration:** Indian space exploration nowadays owes a great deal to the discipline of Indian astronomy. When preparing for space missions, organisations like India's Space Research Organisation (ISRO) look to old astronomical ideas for guidance. A practice with its origins in ancient Indian astronomy, the use of astronomical observations for navigation has had and will have an impact on contemporary space travel.

iv. Preservation and Revival:

- **Textual Preservation:** The careful copying and translation of many old Indian astronomical and mathematical books ensured their survival. The historical contributions of Indian mathematicians and astronomers have been better understood because to the efforts of scholars and organisations to conserve and study these writings.
- **Educational Initiatives:** Contemporary educational programmes use the astronomy and mathematics that were left to us by ancient Indians. We strive to make sure that our students understand and value the scientific legacy that has come before them. Scholars and institutions are still delving into the past, trying to resurrect old mathematical and astronomical techniques.

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v. Recognition and Cultural Pride:

- **National and Cultural Identity:** Ancient Indians take pride in their country and culture because of the work of their mathematicians and astronomers. Appreciating India's scientific legacy and its impact on mathematical and astronomical ideas on a worldwide scale is enhanced by acknowledging these achievements.

CONCLUSION

In conclusion, the massive mathematical and astronomical records that have been discovered in ancient India are evidence of the remarkable intellectual achievements that have been achieved by that society. The Vedic period was essential in the development of early astronomical observations because it included references to the skies and the motions of the heavens in the Rigveda. The Siddhantas, particularly the Brahmasiddhanta and the Surya Siddhanta, were responsible for significant contributions to the advancement of knowledge on the motion of the planets as well as the methodologies for determining eclipses and the placements of the planets.

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